

STAT 100: Week 5

Ricky's Section

Introductions and Attendance

Introduction: Name

Question of the Week: Which best describes you:
Data Visualizer () , Data Wrangler () , or Data
Collector ()?

Important Reminders

Anonymous Feedback

https://docs.google.com/forms/d/e/1FAIpQLSfKv_FGvs0oqm-IvtxKx3Vf6bBzSJE2jamK1gklAzL6NkXE8w/viewform

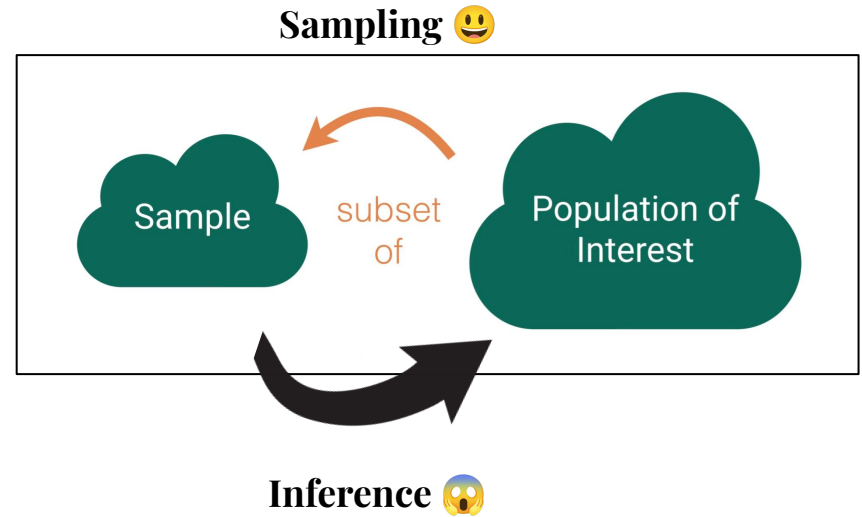
Midterm

- **Written Component:** Wed, Oct. 16 from 6 to 9 PM in SC Hall B
- **Oral Component:** Over Zoom afterwards (10 minute sessions)
- Let me know if you have any questions
- **You all got this!** 😊

Content Review: Week 5

Introduction to Inference

- Last week, we went **from population to sample**. Moving forward, we'll go **from sample to population!**
- Why? Recall the difficulty of obtaining a **census**
- We have data from a **sample** and are interested in concluding something about the **population**



Parameter vs. Statistic

Population parameter:

- Typically **unknown** (what we're interested in finding)
- For **population proportion**, it's denoted as p
 - This is for binary categorical variables
 - There are many other parameters, which we'll soon learn about!
- *Ex: Out of all 67 million viewers of the debate, how many believed Harris won? I don't know!*

Sample statistic:

- **Known**/calculated from the **sample**
- For **sample proportion**, it's denoted as \hat{p}
- *Ex: From my (random) sample of 600 viewers, how many believed Harris won? Let's say it was 300, so $\hat{p} = 0.5$*

A **sample statistic** is a **point estimate** of the **population parameter** (i.e., our best guess, but we could be wrong)

Other Parameters and Statistics

	Response Variable		Numeric Quantity	Sample Statistic	Population Parameter
1 variable	Numerical		Mean	\bar{x}	μ
	Binary Categorical		Proportion	\hat{p}	p
	Response variable	Explanatory Variable	Numeric Quantity	Sample Statistic	Population Parameter
2 variables	Numerical	Binary Categorical	Difference in Means	$\bar{x}_1 - \bar{x}_2$	$\mu_1 - \mu_2$
	Binary Categorical	Binary Categorical	Difference in Proportions	$\hat{p}_1 - \hat{p}_2$	$p_1 - p_2$
	Numerical	Numerical	Correlation	r	ρ

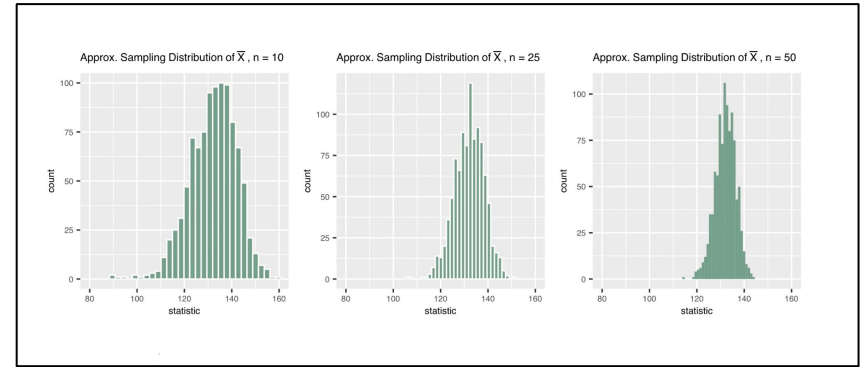
Sampling Variability

- We could've taken a different **sample** of 600 people from the **population** of 67 million viewers
 - The **sample proportion** (probably) would've differed
- **Sampling variability** refers to the **differences in the sample statistic** from sample to sample
 - If we take many samples, how much would the **sample proportion** vary?
 - $\hat{p} = 0.5$ in this sample, but $\hat{p} = 0.4$ in that sample, and so on

Sampling Distribution

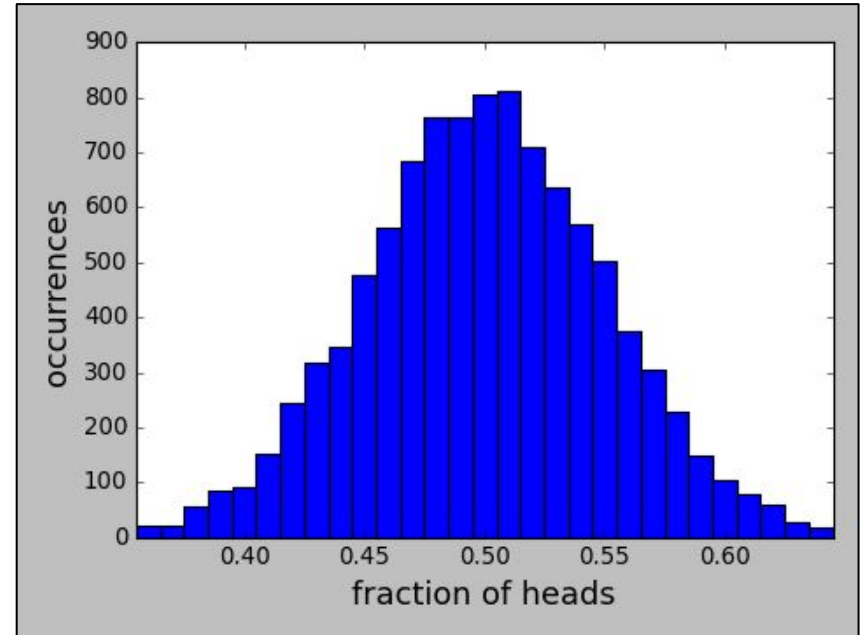
Sampling distribution of a statistic:

- Graph of **sample statistics** from **repeated samples** (requires access to entire population)
- **Center of sampling distribution is population parameter**
- As n , sample size of each rep, increases...
 - **Standard error** (standard deviation of sampling distribution) **decreases** (indicated by less spread)
 - **Sampling distribution** becomes **more bell-shaped and symmetric**



Coin Flips: An Intuition behind Sampling Distributions

- Let's flip a fair coin 10 times and record the proportion of heads
- Will our sample statistic always be 0.5? No!
- The center is the “theoretical” population proportion ($p = 0.5$)
- We're graphing a bunch of sample proportions ($\hat{p}_1 = 0.4$, $\hat{p}_2 = 0.5$, $\hat{p}_3 = 0.6$, ...)



What is the
“problem” with
the sampling
distribution?

Question:

What is the “problem” with the sampling distribution?

To construct a **sampling distribution**, we need access to the entire **population** from which to draw **repeated samples**.

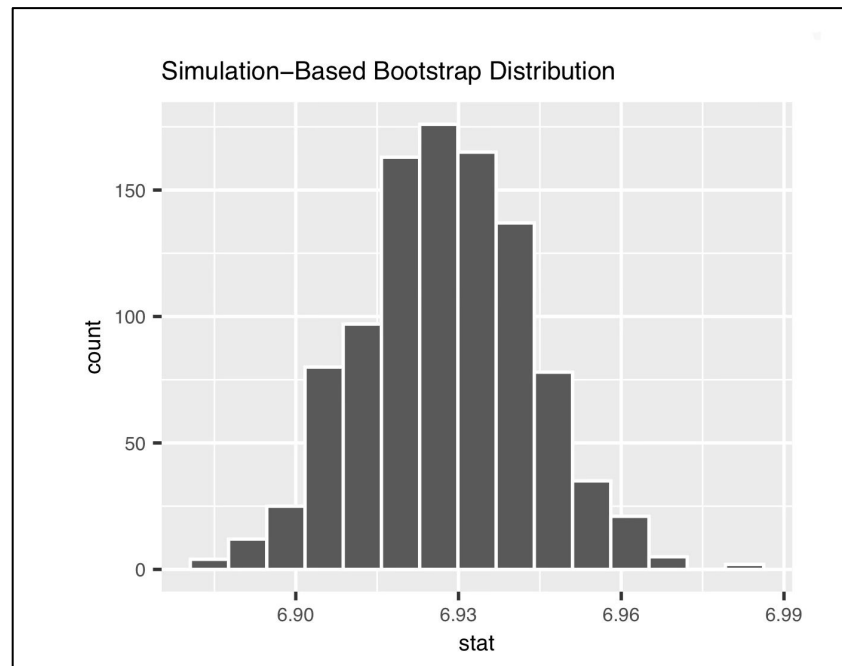
This is not always practical.

Here’s where the **bootstrap distribution** comes in!

Bootstrap Distribution

Bootstrap distribution of a sample statistic:

- **Procedure:** Take a **sample** of size n (with **replacement**) from the **original sample**, compute the statistic on this **bootstrap sample**, and repeat many times to get many **bootstrap statistics** (basically, **sampling the sample**)
 - We no longer need the entire **population**
- **Bootstrap distribution** graphs these **bootstrap statistics**
- **Center of bootstrap distribution** is the **original sample statistic**



Example of Bootstrapping

Population: {100, 250, 75, 30, 50, 75, 100, 300, 120, 55, 80, 90}, $\mu = 110.416\dots$

Original Sample (n = 4): {250, 75, 75, 120}, $\bar{x} = 130$

Bootstrap sample #1: {250, 120, 120, 250}, $\bar{x} = 185$

Bootstrap sample #2: {75, 120, 75, 250}, $\bar{x} = 130$

Bootstrap sample #3: {75, 75, 120, 75}, $\bar{x} = 86.25$

and so on...

Sampling Distribution vs. Bootstrap Distribution

Sampling distribution:

- Requires access to the entire **population**
- Its **center** is the **population parameter**
- Its **spread/standard deviation** is the **standard error**, which we need to compute a CI

Bootstrap distribution:

- Does NOT require access to the entire **population**
 - We only need **1 sample**
- Its **center** is the **sample statistic**
- Its **spread/standard deviation** is a **good estimate for standard error**

Confidence Interval

Confidence interval: Range of **plausible** values (around the **sample statistic**) that may contain the **population parameter**

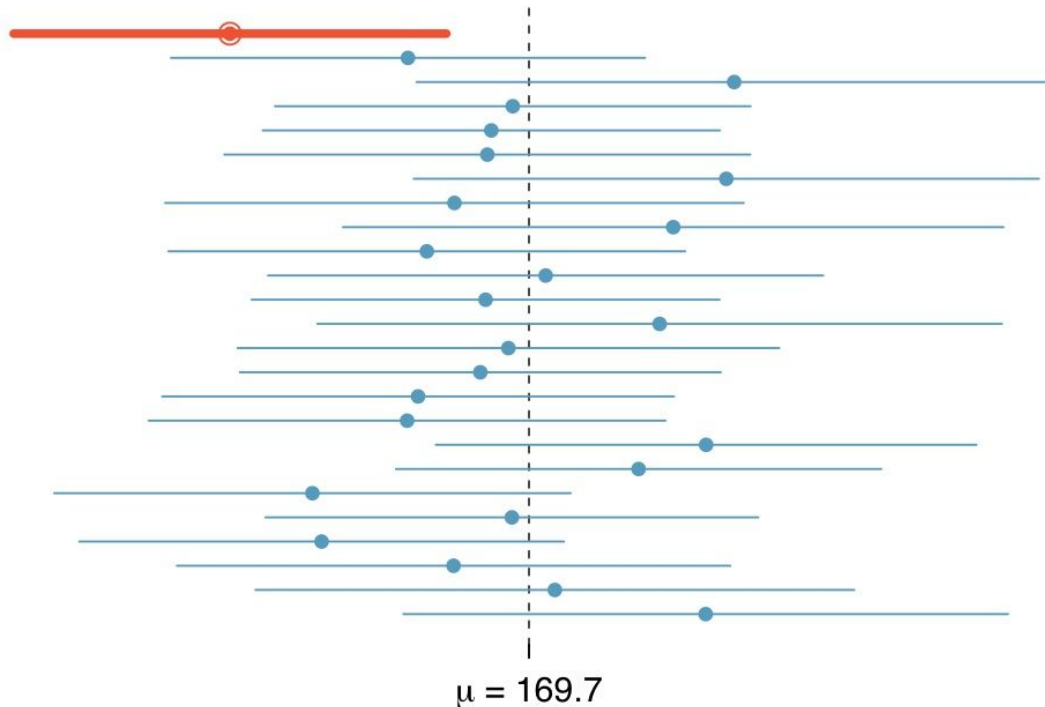
- **SE method**: $CI = \text{statistic} \pm z^* \times (\hat{SE})$
 - z^* is critical value, \hat{SE} is **standard deviation** of bootstrapped statistics (spread of **bootstrap distribution**)
 - *Ex: 95% CI = statistic \pm 1.96(\hat{SE})*
- **Percentile method**: **CI = the middle (CL)% of the bootstrap distribution**
 - CL = confidence level
 - *Ex: 95% CI = the middle 95% of the bootstrap distribution*

Interpreting Confidence Intervals

- “We are {confidence level}% confident that the true {population parameter} lies between {lower bound} and {upper bound}.”
 - Confidence is NOT probability
 - Either the **parameter** in the CI (100% probability) or it's not (0% probability)
 - For a 95% CI, we expect it to succeed (for it to capture the population parameter) **95/100 times**

THE MEANING OF CONFIDENCE...

Twenty-five samples of size $n = 60$ were taken from the 'artificial' population, then a 95% CI for μ was computed based on each sample. Only 1 of these 25 intervals did not contain μ .



Why does the
sampling dist. get
narrower as we
increase n ?

Question:

Why does the sampling dist. get narrower as we increase n ?

n is the **sample size** of each rep.

When n is **small** (e.g., $n = 10$), we're drawing small samples, so a single **outlier** can drastically skew our **sample statistic**. As n increases, **outliers** become less "powerful."

Also, we know when n is the **population**, the **sampling statistic** is just the **population parameter**.

Important Code for Week 5

<https://drive.google.com/file/d/1Rmadk9HC-uP7UopgojoPOtXky81j6lx1/view?usp=sharing>

Questions?

Midterm Review (Weeks 1-5)

Week 2: Data Visualization

- Grammar of graphics: Dataset, geom, aesthetic
- Color palettes: Sequential, diverging, qualitative
- Choosing the right graph

Week 3: Data Wrangling

- **Data joins**: Left, right, inner, full
- **Creating/modifying variables**
- **Grouping/selecting data**
- **Summary statistics**: Mean, median, SD, IQR
- **Handling missing values (NA)**
- **Interpreting code in English**

Week 4: Data Collection

- **Groups**: Sample, census, population
- **Observational study vs. experiment**
- **Two types of bias**: Sampling, nonresponse
- **Four sampling methods**: Simple, systematic, cluster, stratified

Week 5: Simulation-Based Inference

- **Parameter vs. statistic**
- **Distributions**: Sampling, bootstrap
- **Confidence intervals**: Constructing, interpreting

Midterm Tips

- **PLEASE SET A TIMER!** There should be 3 questions in 10 minutes, so try not to “ramble”
- If you haven’t already, make a **study guide**
- **Partial credit** counts
- Remember to **load all relevant libraries**
- **Pace yourself**—if a question is taking too long, move on
- Sign up for **practice oral exams** (usually not 3 questions)

Midterm Practice

Practice 1: Everybody

https://docs.google.com/document/d/1JDJqJlwrJBLWlKcKW0AKYSw_wl4zEviBdOrpme3v8I/edit#heading=h.ewccgla427fy

Practice 2: Person A (Grade Q1, Answer Q2)

https://docs.google.com/document/d/1ukYrohJJBqqYOBBe79gD5uW7h_LUcZ7QEgonaYKox2g/edit?usp=sharing

Practice 2: Person B (Answer Q1, Grade Q2)

<https://docs.google.com/document/d/1B7N4EHJaml05Z8BCDVP8Zr-mb2TiZYtQeKPz-ozy-18/edit?usp=sharing>

P-Set 4

Have a great rest
of your week!